Chapter 11: Outline

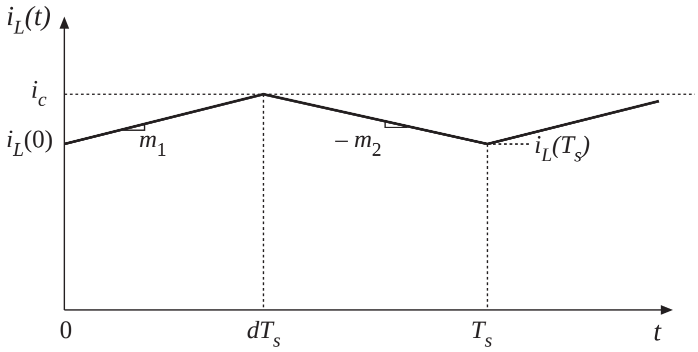
11.1 Oscillation for $D > 0.5$

- The current programmed controller is inherently unstable for $D > 0.5$, regardless of the converter topology
- Controller can be stabilized by addition of an artificial ramp

Objectives of this section:

- Stability analysis
- Describe artificial ramp scheme

Inductor current waveform, CCM



Inductor current slopes m_1 and $-m_2$

buck converter

$$m_1 = \frac{v_g - v}{L} \qquad -m_2 = -\frac{v}{L}$$

boost converter

$$m_1 = \frac{v_g}{L} \qquad -m_2 = \frac{v_g - v}{L}$$

buck–boost converter

$$m_1 = \frac{v_g}{L} \qquad -m_2 = \frac{v}{L}$$

Steady-state inductor current waveform, CPM

First interval:

$$i_L(dT_s) = i_c = i_L(0) + m_1 dT_s$$

Solve for *d*:

$$d = \frac{i_c - i_L(0)}{m_1 T_s}$$

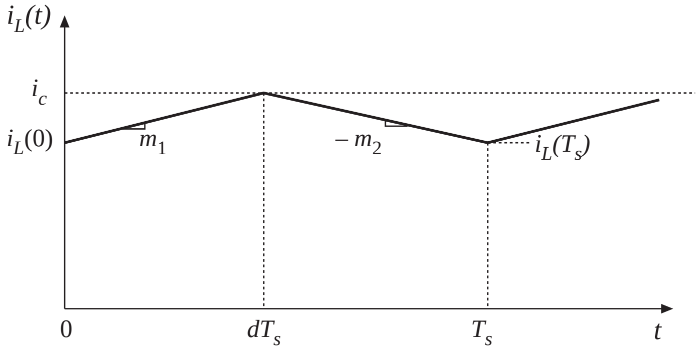
Second interval:

$$i_L(T_s) = i_L(dT_s) - m_2 d'T_s$$
$$= i_L(0) + m_1 dT_s - m_2 d'T_s$$

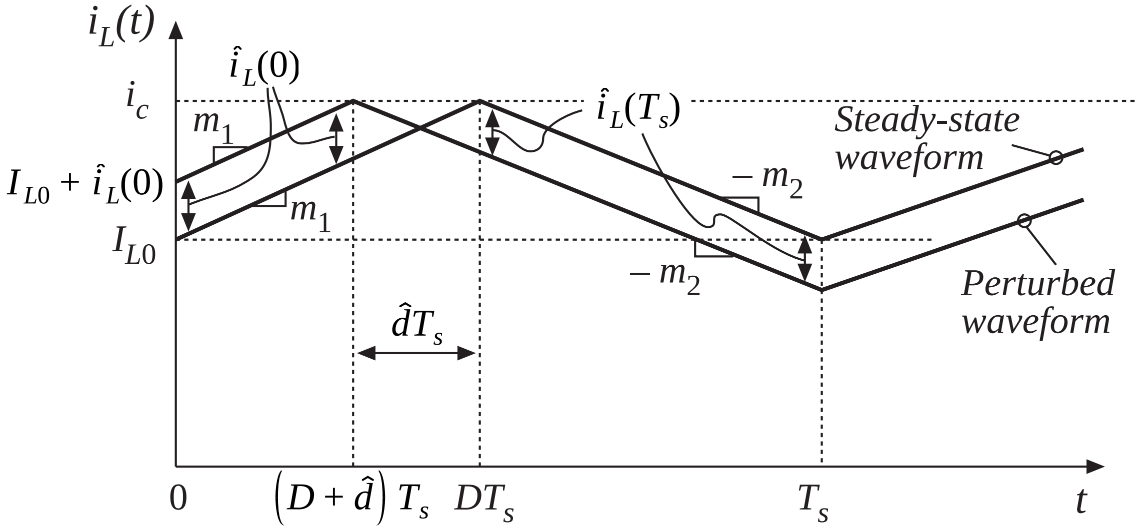
In steady state:

$$0 = M_1 DT_s - M_2 D'T_s$$

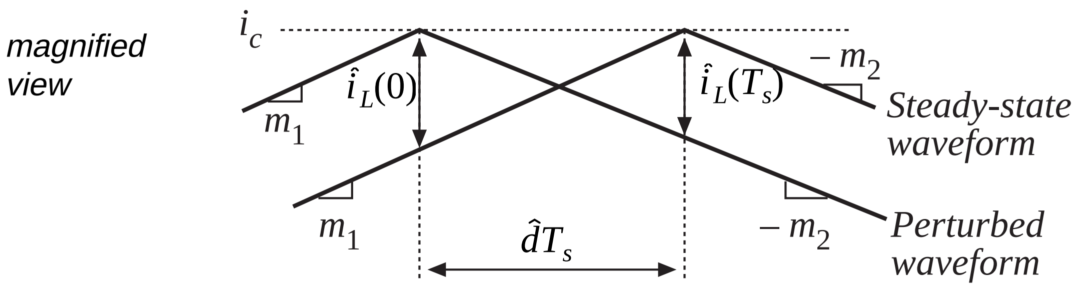
$$\frac{M_2}{M_1} = \frac{D}{D'}$$



Perturbed inductor current waveform



Change in inductor current perturbation over one switching period



$$\hat{i}_L(0) = -\,m_1\hat{d}T_s$$

$$\hat{i}_L(T_s) = m_2\hat{d}T_s$$

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{m_2}{m_1}\right)$$

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)$$

Change in inductor current perturbation over many switching periods

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)$$

$$\hat{i}_L(2T_s) = \hat{i}_L(T_s)\left(-\frac{D}{D'}\right) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)^2$$

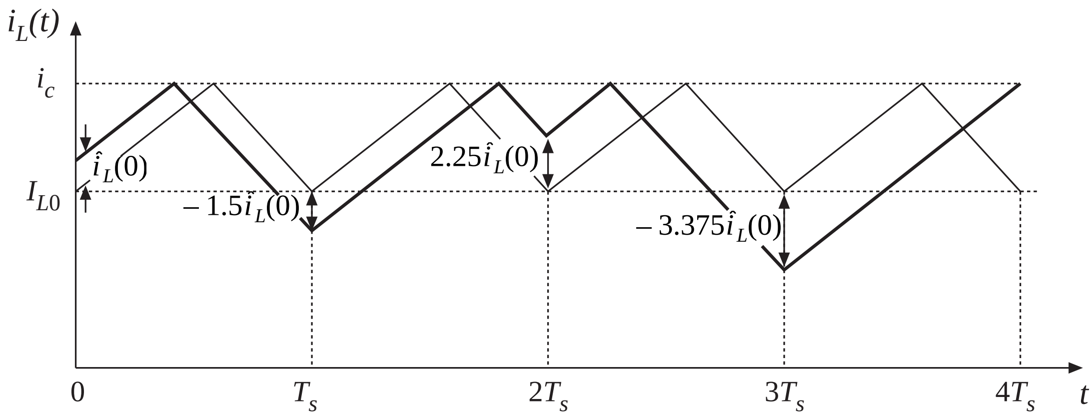
$$\hat{i}_L(nT_s) = \hat{i}_L((n-1)T_s)\left(-\frac{D}{D'}\right) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)^n$$

$$\left|\hat{i}_L(nT_s)\right| \rightarrow \begin{cases} 0 & \text{when } \left|-\frac{D}{D'}\right| < 1 \\ \infty & \text{when } \left|-\frac{D}{D'}\right| > 1 \end{cases}$$

For stability: $D < 0.5$

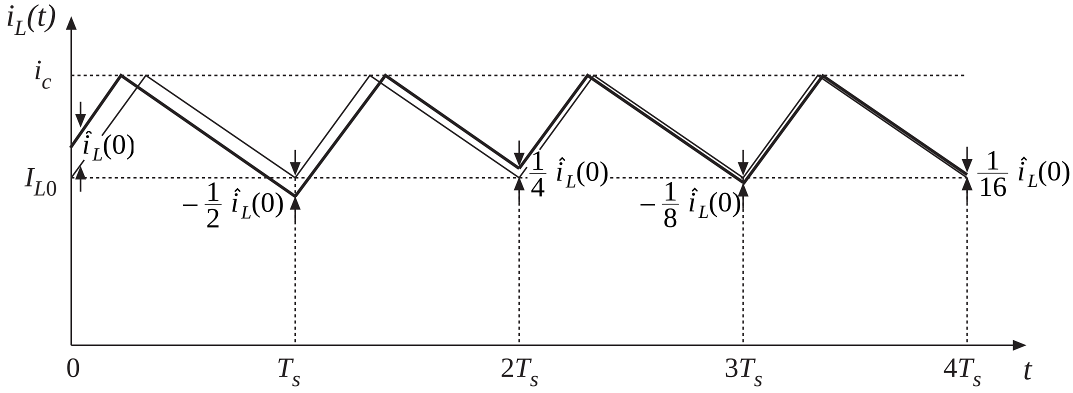
Example: unstable operation for $D = 0.6$

$$\alpha = -\frac{D}{D'} = \left(-\frac{0.6}{0.4}\right) = -1.5$$

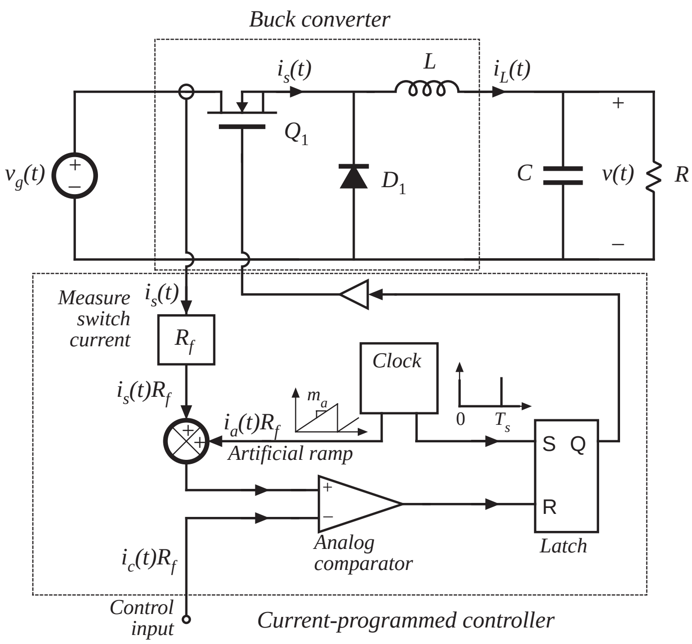


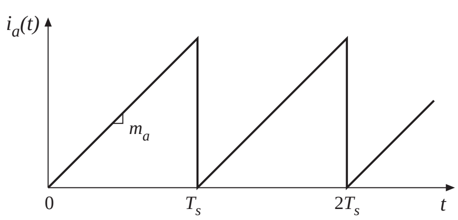
Example: stable operation for $D = 1/3$

$$\alpha = -\frac{D}{D'} = \left(-\frac{1/3}{2/3}\right) = -0.5$$



Stabilization via addition of an artificial ramp to the measured switch current waveform





Now, transistor switches off when

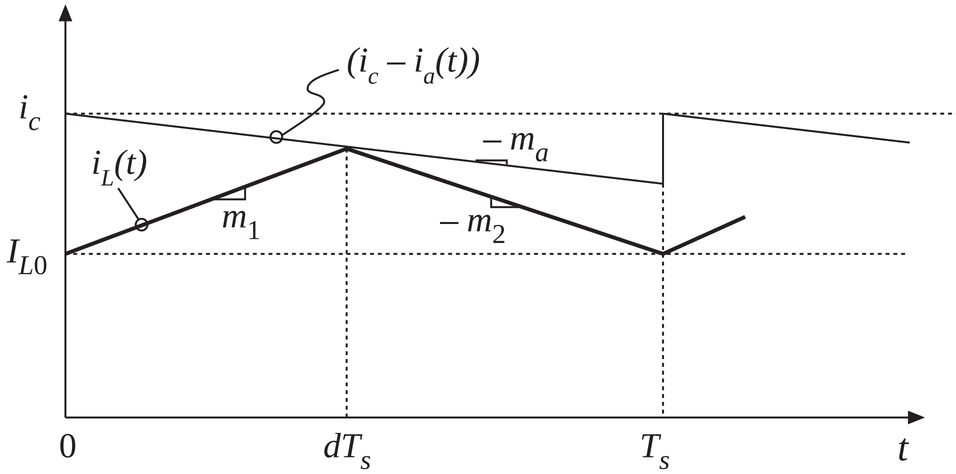
$$i_a(dT_s) + i_L(dT_s) = i_c$$

or,

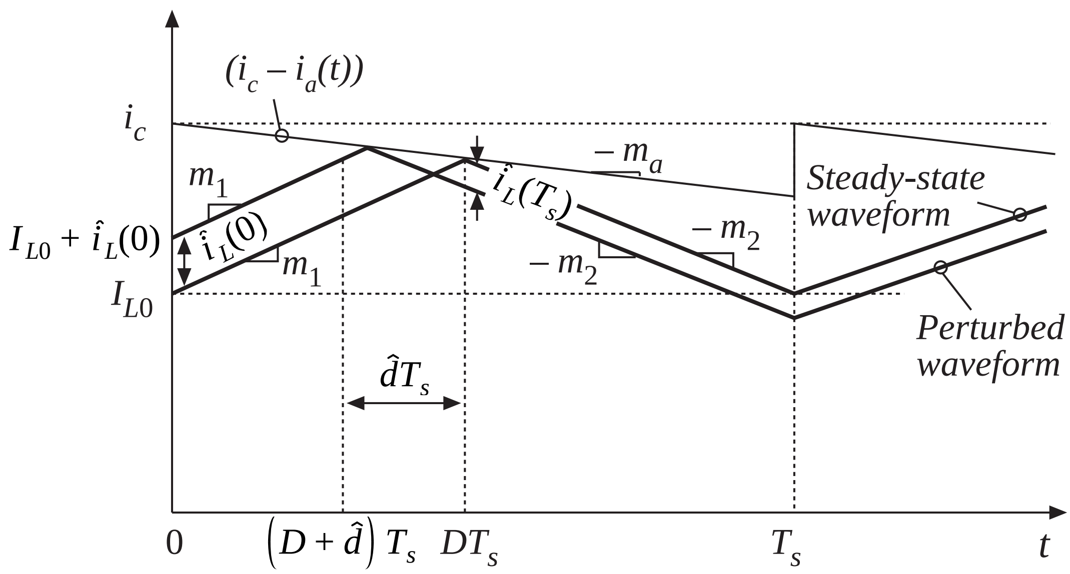
$$i_L(dT_s) = i_c - i_a(dT_s)$$

Steady state waveforms with artificial ramp

$$i_L(dT_s) = i_c - i_a(dT_s)$$



Stability analysis: perturbed waveform



Stability analysis: change in perturbation over complete switching periods

First subinterval:

$$\hat{i}_L(0) = -\hat{d}T_s\left(m_1 + m_a\right)$$

Second subinterval:

$$\hat{i}_L(T_s) = -\hat{d}T_s\left(m_a - m_2\right)$$

Net change over one switching period:

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right)$$

After n switching periods:

$$\hat{i}_L(nT_s) = \hat{i}_L((n-1)T_s)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right) = \hat{i}_L(0)\left(-\frac{m_2 - m_a}{m_1 + m_a}\right)^n = \hat{i}_L(0)\,\alpha^n$$

Characteristic value:

$$\alpha = -\frac{m_2 - m_a}{m_1 + m_a} \qquad \left|\hat{i}_L(nT_s)\right| \to \begin{cases} 0 & \text{when } |\alpha| < 1 \\ \infty & \text{when } |\alpha| > 1 \end{cases}$$

The characteristic value α

$$\alpha = -\frac{1 - \frac{m_a}{m_2}}{\frac{D'}{D} + \frac{m_a}{m_2}}$$

- For stability, require $|\alpha| < 1$
- Buck and buck-boost converters: $m_2 = -v/L$

  So if v is well-regulated, then m_2 is also well-regulated
- A common choice: $m_a = 0.5\, m_2$

  This leads to $\alpha = -1$ at $D = 1$, and $|\alpha| < 1$ for $0 \le D < 1$.

  The minimum α that leads to stability for all D.
- Another common choice: $m_a = m_2$

  This leads to $\alpha = 0$ for $0 \le D < 1$.

  Deadbeat control, finite settling time